Sketch the curve $y = \dfrac{x}{1+x^2}$.

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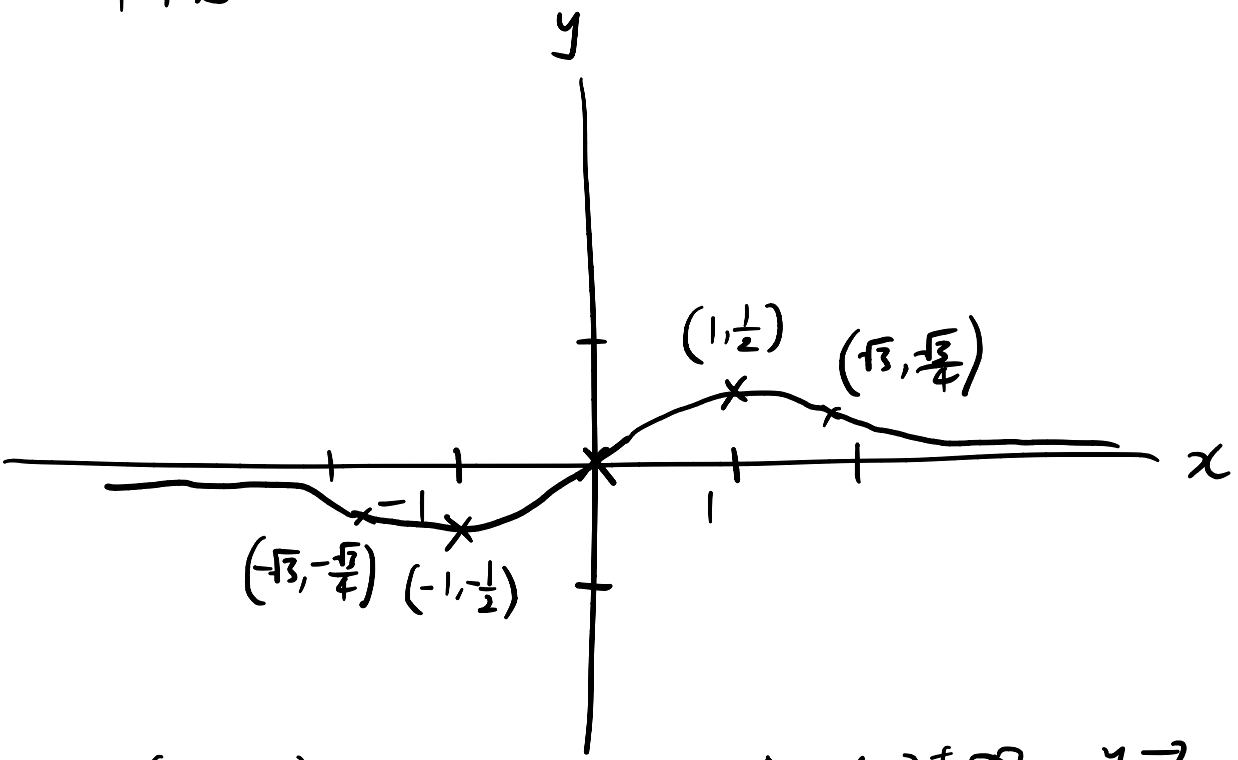
$$y = \frac{x}{1+x^2}$$



$$y' = \frac{1(1+x^2) - x(2x)}{(1+x^2)^2}$$

As $x \to \pm\infty$, $y \to \frac{1}{x^2}$

$\Rightarrow y \to 0$ at both ends

$$= \frac{-x^2+1}{(1+x^2)^2}$$

$y' = 0 \Rightarrow -x^2+1 = 0$

$-(x+1)(x-1) = 0$

$\Rightarrow x = \pm 1$ are critical points.

$y' > 0$ when $-1 < x < 1 \Rightarrow y$ is increasing

$y' < 0$ when $x < -1$ and $x > 1 \Rightarrow y$ is decreasing

$$y'' = \frac{-2x(1+x^2)^2 - (-x^2+1)\,2(1+x^2)\,2x}{(1+x^2)^4}$$

$$= \frac{-2x(1+x^2)(1+x^2-2x^2+2)}{(1+x^2)^4}$$

$$= \frac{-2x(-x^2+3)}{(1+x^2)^3}$$

$$= \frac{2x(x^2-3)}{(1+x^2)^3}$$

$\therefore$ $x = 0, \pm\sqrt{3}$ are inflection points

At $x = -\sqrt{3}$, $y = -\frac{\sqrt{3}}{4}$

$x = \sqrt{3}$, $y = \frac{\sqrt{3}}{4}$

$(0, \sqrt{3})$, y is concave down.

$(-\sqrt{3}, 0)$, y is concave up.